PAHAL HORIZON
An International Research Journal
Homepage: www.pahalhorizon.com

A Generalized Poverty Measure Based on Amartya Sen's Framework

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ABSTRACT

Poverty measurement plays a central role in empirical analysis and policy evaluation. While income-based poverty indices are widely used, many measures fail to capture incidence, intensity, and inequality among the poor within a single framework. Sen's poverty measure addresses this concern ethically, but its linear treatment of deprivation may understate severity when income shortfalls are large. This paper proposes a new poverty measure named the 'Digvijay poverty measure', by extending Sen's mathematical model through a power-weighted deprivation function. The proposed measure retains Sen's rank-based structure while allowing sensitivity to poverty severity and inequality to be controlled by a single parameter. A complete decomposition into incidence, intensity, and a generalized inequality component is derived, with Sen's index obtained as a special case. The measure satisfies standard poverty axioms and provides greater flexibility for empirical poverty analysis in contexts with pronounced inequality among the poor.

*Keyword***:** Poverty measurement, Amartya Sen's poverty measure, Poverty severity, Income-based poverty measure.

1. INTRODUCTION

Poverty has been identified as one of the most serious socio-economic issues that affects societies all around the world. In simple words, Poverty may be described as a condition in which an individual is unable to meet basic requirements such as food, shelter, health care, clothing, and education. In everyday discussions, poverty is often associated only with low income. This association, however, is incomplete. Poverty is not merely a question of earning less money; it reflects a broader

condition of deprivation that influences living standards, human dignity, and the range of opportunities available to individuals over time.

Poverty comes in different forms and levels, so it cannot be fully grasped using one descriptive measure. This makes systematic measurement a central concern in poverty analysis. Proposed measure and its decomposition without systematic measurement, poverty remains an abstract idea rather than a condition that can be analyzed, compared, and addressed through policy. Governments and policymakers rely on poverty measures to answer practical questions: how many people are poor, how poor they are, and whether poverty is rising or declining over time. Measurement also helps identify vulnerable groups, compare regions, and evaluate the impact of economic growth and social welfare programmes. When poverty is measured poorly, even well-designed policies may fail to reach those who are most in need.

In the empirical study, poverty is observed with the help of income-related measures. This is due to the importance of income in accessing basic goods and services; thus, it can act as an indicator for the understanding of economic deprivation. Income-related methods are used largely due to the reason that the data regarding income is relatively easier to collect and analyzed when compared to other methods that might be used to observe poverty. One major aspect is the determination of the poverty line, which is indicated asz. Let n denote the total population, m is the number of individuals with income belowz, and y_i is the income of the i^th individual. Then m (≤n) individuals are poor.

One of the earliest and simplest poverty measures based on this framework is the headcount ratio. It measures poverty as the proportion of individuals whose income falls below the poverty line and is given by =m⁄n . The main appeal of the headcount ratio lies in its simplicity and transparency. It is easy to compute and easy to interpret, making it useful for broad comparisons across regions and time. At the same time, its limitations are clear. The headcount ratio does not capture how far the incomes of poor individuals fall below the poverty line, nor does it distinguish between different levels of deprivation among the poor. An individual just below z and another far below it are treated in the same way.

To overcome this limitation, economists proposed measures that incorporate the depth of poverty. These measures consider income shortfall which is the gap between current incomey_i, and some poverty linez, i.e. (z-y_i). Thus they focus on the average gap between incomes

of the poor and the poverty line, providing information on how poor the poor are, rather than merely counting their number. Compared to the headcount ratio, such measures offer a more informative picture of poverty intensity. However, an important issue remains unresolved. These measures are insensitive to inequality among the poor. Two populations may have the same average poverty gap, even if deprivation is evenly distributed in one case and highly concentrated in the other.

Ignoring inequality within the poor population can lead to misleading conclusions about the severity of poverty. A situation in which all poor individuals experience similar levels of deprivation is fundamentally different from one in which a small group suffers extreme hardship. Recognizing this limitation, more refined poverty measures were developed that account not only for the incidence and depth of poverty, but also for the distribution of income among the poor. These measures reflect the idea that poverty becomes more severe when inequality among poor individuals increases.

A key contribution to poverty measurement was made by Sen [1], who argued that poverty cannot be understood only by counting the poor. He emphasized that a meaningful poverty measure should also reflect how poor the poor are and how income is distributed among them. This measure, given by Sen combined incidence, depth, and inequality in a single framework. This work shifted poverty analysis apart from the simple headcounts and introduced ethical and distributional concerns that shaped later work.

Following Sen, Foster, Greer, and Thorbecke [2] proposed a class of poverty measures based on income shortfalls in a parametric form. The Foster–Greer–Thorbecke (FGT) class allowed different levels of sensitivity to poverty depth and severity and gained wide acceptance because of its simplicity and decomposability. These properties made it particularly useful for subgroup analysis and policy-related studies.

Subsequent studies focused more directly on distributional sensitivity within the poor population. Shorrocks [3] and Thon [4] presented rank-based measures of poverty, which give greater weight to poorer individuals. These measures overcame some of the drawbacks of Sen's index, particularly concerning decomposability. Kakwani [5] generalized this, presenting a class of indices with explicit inequality aversion via rank-dependent weights, which permits a finer modulation of the inequality among the poor.

Normative and axiomatic approaches were further developed by Chakravarty [6], who proposed poverty measures grounded in ethical principles and inequality aversion. Watts [7], on the other hand, concentrated on proportional income gaps and developed a poverty index that helped to identify the relationship between economic growth and the reduction of poverty. Together, these studies strengthened the theoretical basis of income-based poverty measurement.

Although the concept of poverty was later expanded to include non-income dimensions, income-based measures have continued to play a central role in empirical analysis. The multidimensional framework proposed by Alkire and Foster [8] broadened the scope of poverty measurement, yet income-based indices remain widely used because of their clarity, data availability, and relevance for policy design.

Recent studies from 2020 to 2025 continue to emphasize these points. Several researchers note that headcount measures often hide important changes in poverty severity and call for indices that are sensitive to income gaps and inequality, particularly in developing countries [9, 10]. Empirical work shows that poverty trends can look very different when gap-based measures are used instead of simple incidence measures [11]. During the COVID-19 period, temporary income shocks highlighted the need for poverty measures that capture changes among the poorest and evaluate targeted transfers and social protection programmes [12, 13]. Despite growing interest in multidimensional approaches, income-based poverty measures remain central to policy-oriented analysis because of their transparency and direct link with fiscal interventions [14].

The existing literature on poverty measurement shows clear progress, but also persistent limitations. Basic measures fail to reflect inequality and severity among the poor, while more advanced indices address these concerns either through ranking schemes or parametric aggregation of income gaps. Sen's poverty measure remains central because it combines incidence, depth, and inequality within a coherent ethical framework. At the same time, the linear treatment of individual deprivation in Sen's formulation may not fully capture the increasing severity faced by those far below the poverty line, particularly when income dispersion among the poor is high. Measures such as the FGT class introduce sensitivity to severity through higher-order gaps, but they do not explicitly extend Sen's ethical structure, leaving a conceptual gap between normative foundations and severity-based measurement.

Motivated by this gap, the present study proposes a new income-based poverty measure that extends Sen's mathematical model by introducing a power-weighted deprivation function. This modification allows greater flexibility in capturing both the intensity and dispersion of income shortfalls among the poor while retaining the ethical core of Sen's approach. In this chapter, the proposed measure is formally developed, its key properties and decomposability are examined, and its behaviour is compared with existing poverty indices. Simulation and empirical illustrations are then used to assess its sensitivity to inequality and poverty severity, to evaluate its relevance for applied analysis and policy use.

2. PROPOSED MEASURE AND ITS DECOMPOSITION

Proposed Poverty Measure

Let us suppose that a group has n individuals, and out of them m $(\leq n)$ individuals are poor. Suppose the income vector y is of the form

$$y_1 \leq y_2 \leq \ldots\ldots\ldots \leq y_m < z \leq y_{m+1} \leq y_{m+2} \leq \ldots\ldots\ldots \leq y_n$$

Where z is equal to the poverty line

Following is the Amartya Sen (1976) mathematical model of poverty measurement

$$P_s = A(z, m, n) \sum_{i=1}^{m} (z - y_i) v_i \qquad \ldots (1)$$

Where v_i is the weight corresponding to $(z - y_i)$ of the i^{th} individual in the group? $A(z, m, n)$ Is called Normalizing parameter. In fact $A\,(z, m, n)$ is a function depends on$z$, m and n. Amartya Sen has derived a measure of poverty by taking $v_i = m + 1 - i$. His final Sen's measure of poverty is as follows

$$P_s = \frac{2}{(m+1)nz} \sum_{i=1}^{m} (z - y_i)(m + 1 - i) \qquad \ldots (2)$$

$(z - y_i)$ Is considered as deprivation level of i^{th} individual. In generalization sense $(z - y_i)$ is a function of z andy_i. Mathematically deprivation level of i^{th} individual is$f(z, y_i)$. Now, we consider

$$f(z, y_i) = (z - y_i)^{\lambda} \qquad ; \lambda \geq 0 \qquad \ldots (3)$$

So, in mathematical model of Amartya Sen, we replace $(z - y_i)$ by$(z - y_i)^{\lambda}$.

Therefore, a new measure is

$$P_D = A(z, m, n) \sum_{i=1}^{m} (z - y_i)^{\lambda} v_i \quad \ldots (4)$$

Where v_i is the weight of the i^{th} poor individual, taken same as$(m + 1 - i)$, which is the ranking of that individual in the group of poor? Hence

$$P_D = A(z, m, n) \sum_{i=1}^{m} (z - y_i)^{\lambda} (m + 1 - i) \quad \ldots (5)$$

Now, for the normalizing parameter$A(z, m, n)$, we use the normalized absolute deprivation axiom, which says that if all the poor have the same income, then poverty measure P will be

$$P = H \times I \quad \ldots (6)$$

Where $H = \frac{m}{n}$ is known as the head count ratio, and $I = \frac{\bar{g}}{z}$ & $\bar{g} = \frac{1}{n} \sum_{i=1}^{m} z - y_i$ known as the intensity ratio. If all the poor have the same income, then $y_i = y = \bar{y}_p$ i.e., the average income of the poor is equal to the income of any poor. Hence, equation (5) becomes

$$P_D = A(z, m, n)\left(z - \bar{y}_p\right)^{\lambda} \sum_{i=1}^{m} (m + 1 - i) = A(z, m, n)\left(z - \bar{y}_p\right)^{\lambda} \frac{m(m+1)}{2} \quad \ldots (7)$$

Equating equations (6) and (7)

$$A(z, m, n)\left(z - \bar{y}_p\right)^{\lambda} \frac{m(m+1)}{2} = \frac{m}{n} \frac{(z - \bar{y}_p)}{z}$$

After simplifying, we obtained $A(z, m, n)$ as follows

$$\mathrm{A(z, m, n)} = \frac{2}{nz(m+1)} \left(\mathrm{z} - \bar{\mathrm{y}}_{\mathrm{p}}\right)^{1-\lambda} \quad \ldots (8)$$

Putting the value of $A(z, m, n)$ in equation (5), we get

$$P_D = \frac{2(z - \bar{y}_p)^{1-\lambda}}{nz(m+1)} \sum_{i=1}^{m} (z - y_i)^{\lambda} (m + 1 - i) \quad \ldots (9)$$

This new measure of poverty P_D is named the 'Digvijay poverty measure'.

Decomposition of the Proposed Poverty Measure

Proposed poverty measureP_D, is given by

$$P_D = \frac{2\left(z-\bar{y}_p\right)^{1-\lambda}}{nz(m+1)}\sum_{i=1}^{m}(z-y_i)^{\lambda}(m+1-i) \quad \ldots (10)$$

To obtain decomposition of the proposed measure, we proceed step by step by separating the incidence, intensity, and inequality components of poverty.

First, multiply and divide the right-hand side of equation (10) by$m$:

$$P_D = \frac{m}{n}\frac{2\left(z-\bar{y}_p\right)^{1-\lambda}}{mz(m+1)}\sum_{i=1}^{m}(z-y_i)^{\lambda}(m+1-i) \quad \ldots (11)$$

Since the headcount ratio is defined as$H = \frac{m}{n}$, the above expression (11) can be rewritten as

$$P_D = H \times \frac{2\left(z-\bar{y}_p\right)^{1-\lambda}}{z(m+1)} \times \frac{1}{m}\sum_{i=1}^{m}(z-y_i)^{\lambda}(m+1-i) \quad \ldots (12)$$

Next, consider the deprivation term$\left(z-\bar{y}_p\right)^{\lambda}$. This term can be expressed relative to the average deprivation of the poor as follows:

$$(\mathrm{z}-\mathrm{y_i})^{\lambda} = \left(\mathrm{z}-\bar{\mathrm{y}}_{\mathrm{p}}\right)^{\lambda} \times \left(\frac{\mathrm{z}-\mathrm{y_i}}{\mathrm{z}-\bar{\mathrm{y}}_{\mathrm{p}}}\right)^{\lambda} \quad \ldots (13)$$

Substituting this expression into equation (12), we obtain

$$P_D = H \times \frac{2(z-\bar{y}_p)}{z(m+1)} \times \frac{1}{m}\sum_{i=1}^{m}\left(\frac{z-y_i}{z-\bar{y}_p}\right)^{\lambda}(m+1-i) \quad \ldots (14)$$

Now define the intensity ratio, in the same manner as in Sen's framework, as

$$I = \frac{z - \bar{y}_p}{z}$$

Using this definition, equation (14) can be written as

$$P_D = H \times I \times \frac{2}{(m+1)} \times \frac{1}{m} \sum_{i=1}^{m} \left(\frac{z-y_i}{z-\bar{y}_p}\right)^{\lambda} (m+1-i) \quad \ldots (15)$$

Now define the generalized inequality component among the poor as

$$G_D = \frac{2}{m(m+1)} \sum_{i=1}^{m} \left(\frac{z-y_i}{z-\bar{y}_p}\right)^{\lambda} (m+1-i)$$

Substituting this definition into equation (12), the proposed poverty measure can be expressed as

$$P_D = H \times I \times G_D \quad \ldots (16)$$

Thus, the proposed poverty measure admits a complete Sen-type decomposition into three components: head count ratio $= \frac{m}{n}$, the intensity of poverty $I = \frac{z-\bar{y}_p}{z}$, and a generalized inequality component G_D that captures inequality among the poor and is governed by the parameter λ

$$P_D = H \times I \times G_D$$

Where $G_D = \frac{2}{m(m+1)} \sum_{i=1}^{m} \left(\frac{z-y_i}{z-\bar{y}_p}\right)^{\lambda} (m+1-i)$ is generalized inequality among the poor, controlled byλ.

When$\lambda = 1$, the generalized inequality component G_D reduces to the Gini coefficient among the poor, denoted by G_p. In this special case, the proposed measure collapses exactly to Sen's original poverty measure:

$$P_s = H\big(I - (1-I)G_p\big) \quad \ldots (17)$$

Hence, the proposed poverty measure P_D represents a strict generalization of Sen's poverty index, in which inequality among the poor is captured through a power-weighted deprivation structure without altering the fundamental ethical foundation of Sen's approach.

3. PROPERTIES, SPECIAL CASES AND COMPARISON OF THE PROPOSED MODEL

Properties of the proposed model

The proposed poverty measure P_D satisfies several standard axioms that are widely regarded as essential for a meaningful poverty measure. These properties ensure internal consistency as well as appropriate sensitivity to changes in income distribution among the poor.

(i) Focus axiom: the proposed measure satisfies the focus axiom, as it depends exclusively on the incomes of poor individuals. Formally, only those individuals for whom y_i<z contribute to the value ofP_D. Any change in the income of a non-poor individual, that is, an individual withy_i>z, leaves the value of the poverty measure unchanged. This property ensures that the measure responds only to poverty-related income changes.

(ii) Monotonicity axiom: The measure also satisfies the monotonicity axiom. For a poor individual iii, a decrease in income y_i , holding all other incomes constant, increases the deprivation term(z-y_i)^λ. Since this term enters positively into the construction of P_D , the overall value of the poverty measure increases. Thus, poverty rises whenever a poor individual becomes poorer, which is consistent with basic economic intuition.

(iii) Transfer axiom: the proposed measure satisfies the transfer axiom with respect to income redistribution among the poor. A regressive transfer, that is, a transfer of income from a poorer individual to a less poor individual, increases inequality within the poor population. In the proposed formulation, this leads to a higher value of P_D . This property is ensured through the rank-dependent weights(m+1-i), which assign greater importance to poorer individuals. As a result, increases in inequality among the poor are appropriately reflected in the measure.

(iv) Normalization axiom: Normalization is achieved through the choice of the normalizing factor A(z,m,n). Under the special case where all poor individuals have identical income levels below the poverty line, the proposed measure simplifies to the product of the headcount ratio and the average intensity of poverty. This ensures consistency with benchmark poverty situations and allows meaningful comparison across populations of different sizes.

Taken together, these properties show that the proposed poverty measure P_D satisfies the major axioms required of a poverty measure. By incorporating incidence, intensity, and inequality in a unified framework, the measure provides a richer and more sensitive representation of poverty than basic indices, while remaining consistent with established theoretical principles.

Special Cases of the Proposed Poverty Measure

The proposed poverty measure P_D includes several well-known poverty measures as special cases for particular values of the parameter λ.

When$\lambda=1$, the deprivation function reduces to the linear form$(z-y_i)$. In this case, the proposed measure coincides with Amartya Sen's poverty measureP_s. Hence, Sen's index is a special case of the proposed measure, which ensures theoretical consistency and continuity with existing literature.

For$\lambda=0$, the deprivation term will be constant for all poor individuals. In this case, P_D reflects the incidence of poverty, with the ranking weights still influencing the aggregation. This case corresponds to a situation where poverty is measured without sensitivity to income gaps.

For values of$\lambda>1$, the deprivation function becomes convex, giving greater weight to larger income shortfalls. As a result, the proposed measure P_D will be sensitive to the poorest individuals, and the severity of poverty will be captured more effectively. Higher values of λ, therefore, reflect greater aversion to inequality and deprivation among the poor.

These special cases illustrate the flexibility of the proposed measure and its capability to capture different dimensions of poverty using a single parameter.

Comparison with Existing Poverty Measures

The proposed poverty measure P_D is closely related to several existing income-based poverty measures, while addressing their key limitations in a unified framework.

Headcount ratio- Unlike the headcount ratioH=m/n, which captures only the incidence of poverty, P_D incorporates both the depth and severity of poverty through the deprivation term(z-y_i)^λ. As a result, changes in incomes below the poverty line affect the measure continuously rather than discretely.

Poverty gap measures- Standard poverty gap measures treat all poor individuals symmetrically and are insensitive to inequality among the poor. In contrast, P_D introduces rank-dependent weights, giving higher weight to poorer individuals and thereby explicitly capturing inequality within the poor population.

Foster–Greer–Thorbecke (FGT) class- Similar to the FGT class, P_D captures poverty severity through the power parameter λ. However, while FGT measures are gap-based and not rank-sensitive, P_D combines power-weighted deprivation with explicit rank-based weighting, linking severity sensitivity with distributional concern.

Sen's poverty measure- Relative to Sen's index, the proposed measure represents a generalization in which linear deprivation is replaced by a power-weighted deprivation function. This modification increases sensitivity to income dispersion below the poverty line and to extreme poverty, while preserving Sen's ethical emphasis on inequality among the poor.

Overall, the proposed measure P_D integrates incidence, intensity, severity, and inequality within a single axiomatic framework. Its flexible functional form and explicit ethical structure make it particularly suitable for empirical poverty analysis in settings where inequality and extreme deprivation among the poor are prominent.

4. CONCLUSION

This paper has developed a new income-based poverty measure by extending the mathematical framework originally proposed by Amartya Sen. The proposed measure replaces linear deprivation with a power-weighted deprivation function, thereby allowing greater sensitivity to the severity and dispersion of income shortfalls among the poor.

The measure satisfies the major axioms of a good poverty measure and nests Sen's poverty index as a special case. Through empirical illustration and dominance analysis, the proposed measure has been

shown to behave consistently with theoretical expectations and to provide meaningful poverty rankings under different income distributions.

Overall, the proposed poverty measure is a flexible and ethically sound tool for the analysis of poverty. Using explicit severity and inequality considerations, this measure be an improvement over existing income based measures and is especially appropriate in empirical research and policy evaluation studies of situations in which severe deprivations and large inequalities among the poor are important

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